**Barron’s Let’s Review Regents – Algebra I**

# Chapter 3: Quadratic Equations

## 3.1 Solving Quadratic Equations by Taking the Square Root of Both Sides of the Equation

An equation that has a variable raised to the second power is called a quadratic equation. Quadratic equations require more advanced techniques to solve. The simplest quadratic equations to solve are the ones where there is no x-term, like:   
x2 = 25 or x2 + 2 = 38.

Examples:   
x2 = 9  
x2 + 5x = 24  
x2 + 5x + 6 = 0

### One-Step Quadratic Equations

If a quadratic equation is in the form x2 = c, the exponent can be eliminated by taking the square root of both sides. There are two answers, generally, since positive times positive is a positive whereas negative times negative also equals a positive.

**Example 1**

Solve for all values of x that satisfy the equation of   
x2 = 25.

x2 = 25  
 =   
x = , x = 5 or x = -5

**Example 2**

x2 = 26  
 =   
x =

### Two-Step Quadratic Equations with Constants

**Example 3**

x2 + 5 = 54  
-5 = -5  
x2 = 49  
 = =

**Example 4**

Solve for all values of x that satisfy the equation   
x2 + 9 = 36.

x2 + 9 = 36  
-9 = -9  
x2 = 27  
 =   
x = , = +

**Example 5**

Solve for all values of x that satisfy the equation:  
(x + 1)2 = 25  
 =   
x + 1 =   
-1 = -1  
x =   
x = 4, x = -6

**Example 6**

Solve for all values of x that satisfy the equation:  
(x + 3)2 = 17  
 = 17  
x + 3 =   
-3 = -3  
x = -3   
x = -3 + or x = -3 -

**Example 7**

Solve for all values of x that satisfy the equation:  
x2 + 6x + 9 = 17

x2 + 6x + 9 = 17  
Factors: **(3,3)**, (-3,-3), (1,9), (-1, -9)  
(x + 3)2 = 17  
x + 2 =   
-2 = -2  
x = -2 + , or x = -2 -

### Two-Step Quadratic Equations with Coefficients

Solve 2x2 = 50.

**Example 8**

Solve for x in terms of a and b in the equation ax2 = b.

ax2 = b

**Example 9**

The formula for the volume of a cylinder is . Solve this equation for r in terms of V, h, and .

Since r represents the radius of the cylinder, the negative answer can be disregarded.

### Solving Square Root Equations

x = 9

x – 2 = 25  
2 = 2  
x = 27

If the square root sign is not already isolated, first isolate it with algebra.

-3 = -3  
x = 25

### Check Your Understanding of Section 3.1

1. Multiple Choice
2. Find all solutions to x2 = 36.  
   **(4) 6, -6**
3. Find all solutions to x2 = 37.  
   **(1)**
4. Find all solutions to x2 - 9 = 40.  
   **(3) 7, -7**
5. Find all solutions to (x + 2)2 = 64.  
   **(2) 6, -10**
6. Find all solutions to (x – 3)2 = 13.  
   **(3)**
7. Find all solutions to (x + 2)2 = 17.  
   **(4)**
8. Find all solutions to 3x2 = 108.  
   x2 = 36  
   (4) 6, -6
9. Find all solutions to 4x2 + 3 = 103.  
   x2 = 25,   
   **(3) 5, -5**
10. Solve for x in terms of c and d: cx2 = d  
    **(2)**
11. Solve for x in terms of g and h:  
    (x + g)2 = h  
    (1)
12. Show how you arrived at your answers.
13. Find all solutions to the equation x2 = 15, rounded to the nearest hundredth.  
    Use calculator:
14. The left-hand side of the equation   
    x2 + 10x + 25 = 64 can be factored into   
    (x + 5)2. Show how you can find the two solutions to this equation by first factoring the left hand side of the equation.  
    -5 = -5
15. If the area of the largest square in this diagram (composed of the two smaller squares and two rectangles) is 49 square units, what is the length of segment x?  
    -2 = -2  
    x = 5  
    Negative results can be disregarded because the length must be a positive number.
16. Diana solves the equation (x + 4)2 = 97 as follows:  
    (x + 4)2 = 97  
    x2 + 16 = 97  
    Wrong approach, incorrect squaring.
17. Find the solution to the equation (x + 1)3 = 8.  
    (x + 1)3 = 8  
    x + 1 = 2  
    x = 1

## 3.2 Solving Quadratic Equations by Guess and Check

There are several ways to solve a quadratic equation like x2 + 10x = 39. For some examples, when the answer is an integer, it is possible to find the answer through guess and check.

Factors: **(-3,13),** (3, -13)

x2 + 10x – 39 = 0  
(x -3)(x+13) = 0  
x = 3, -13

Guess and check is useful if the quadratic equation is a multiple choice question so there are only four things to check.

**Example 1**

Which of the four choices is a solution to the equation x2 + 5x -3 = 33?  
42 + (5)(4) – 3 = 16 + 20 – 3 = 33. Correct.  
**(3) 4**

**Math Facts**

The solutions to a quadratic equation are sometimes called the *roots* or the *zeros* of the equation.

**Example 2**

Which of the four choices is a root of the equation  
x2 – 6x + 4 = 0?

This requires a calculator.

22.391824 - 28.392 + 4 = - 2  
  
27.41674321 - 31.4166 + 4 = 0.0001

Answer:

### Check Your Understanding of Section 3.2

1. Multiple Choice

For each question, use your calculator to check which answer satisfies the quadratic equation:

1. x2 – 8x + 15 = 0  
   (3)(3) – (8)(3) + 15 = 9 – 24 + 15 = 0  
   **(1) 3**
2. x2 – 2x – 8 = 0  
   (1)(1) – (2)(1) – 8 = 1 – 2 – 8 = -9  
   (2)(2) – (2)(2) – 8 = 4 – 4 – 8 = -8  
   (3)(3) – (2)(3) – 8 = 9 – 6 – 8 = -5  
   (4)(4) – (2)(4) – 8 = 16 – 8 – 8 = 0  
     
   **(4) 4**
3. x2 + 5x – 6 = 0  
   (6)(6) + (5)(6) – 6 = 36 + 30 – 6 = 60  
   (-6)(-6) + (5)(-6) – 6 = 36 – 30 – 6 = 0  
     
   **(2) -6**
4. 2x2 + 5x – 3 = 0  
   (2)(1/4)(1/4) + (5)(1/4) – 3 = (2/16) + (5/4) – 3 = (1/8) + (10/8) – 3 = -1/8  
   (2)(1/3)(1/3) + (5)(1/3) – 3 = (2/9) + (5/3) – 3 = (2/9) + (15/9) – 3 = (17/9) – 3 = -10/9  
   (2)(1/2)(1/2) + (5)(1/2) – 3 = (2/4) + (5/2) – 3 = (1/2) + (5/2) – 3 = (6/2) – 3 = 3 – 3 = 0  
   **(3) (1/2)**
5. x2 + 7x = 30  
   (1)(1) + (7)(1) = 8 30  
   (3)(3) + (7)(3) = 9 + 21 = 30  
     
   **(2) 3**
6. x2 – 2x – 2 = 0  
   6.472 – 2 = 1.9997  
   **(4)**
7. x2 + 5x = 6  
     
   (-6)(-7) + (5)(-7) = 42 – 35 = 7 6  
   (-6)(-6) + (5)(-6) = 36 -30 = 6  
   **(2) -6**
8. x2 + 10x + 13 = 0  
   -5 + = -1.5359  
   -4 + = -0.5359  
   -3 + = 0.4641  
   -2 + = 1.4641  
   **(1) -5 +**
9. x2 + 4x = 25  
   x2 + 4x – 25 = 0  
   **(2) -2 -**
10. x2 – 12x – 14 = 0  
    **(4) 6 +**

## 3.3 Solving Quadratic Equations by Completing the Square

Completing the square is a technique for solving quadratic equations that relies on the concept of a perfect square trinomial. It is not the quickest method to solve certain quadratic equations, but it can be used for all quadratic equations.

A perfect square trinomial, like x2 + 6x + 9, is one where the constant term, the 9, is equal to the square of half the coefficient of the x, in this case, the 6.

If the left-hand side of a quadratic equation is already a perfect square, you can factor it and solve using the technique in Section 3.1, Example 7.

If the left-hand side is not a perfect square trinomial, it is possible to make it into one by adding to both sides of the equation. For the equation  
 , you look at the left-hand side and ask, “What constant would make the left-hand side the equation into a perfect square trinomial?” Since is 25, would be a perfect square trinomial.

Add 25 to both sides and complete the question:

You can check these answers by first substituting 3 for both x’s in the equation and then substituting -13 for both x’s in the equation and making sure that they evaluate to 39.

**Example 1**

What are the roots of the equation x2 + 12x = 28?

Example 2

What are the roots of the equation x2 + 4x - 21

**Rational and Irrational Numbers**

A rational number is one whose decimal expansion either terminates or has a repeating pattern. For example, ½ = 0.5 and 4/7 = 0.571428571428 … are rational numbers. An irrational number does not repeat or have a repeating pattern. For example, .’’

**Combining Rational Numbers with Rational Numbers**

When you add, subtract, multiply, or divide a rational number by a rational number, the result is always another rational number.

**Combining Rational Numbers with Irrational Numbers**

When you combine irrational numbers with non-zero rational numbers, the result is always an irrational number.

**Combining Irrational Numbers with Irrational Numbers**

When you combine an irrational number with another irrational number, the result is usually, but not always, an irrational number.

However, it is possible to combine two irrational numbers and get a rational result.

### Check Your Understanding of Section 3.3

1. Multiple Choice
2. x2 + 12x + c is a perfect square trinomial. What is the value of c?  
   (1) 36
3. x2 + bx + 49 is a perfect square trinomial. What is one possible value of b?  
   (2) 14 (also -14)
4. Use completing the square to find both solutions for x in the equation x2 + 8x + 16 = 9.  
   (x + 4)2 = 9  
   x + 4 =   
   x = = -1, -7  
   (1) -1, -7
5. Use completing the square to find both solutions for x in the equation x2 + 10x = 24.  
   x2 + 10x = 24  
   25 = 25  
   x2 + 10x + 25 = 49  
   (x + 5)2 = 49  
   x + 5 = x = -5 = 2, -12  
   (4) 2, -12
6. Use completing the square to find both solutions for x in the equation x2 – 18x = 40.  
     
   x2 – 18x = 40  
   81=81  
   x2 – 18x +81 = 121  
   (x – 9)2 = 121  
   x – 9 =   
   x = 9 = -2, 20  
   (2) -2, 20
7. What is one solution to the equation   
   x2 – 6x = 8?  
   9 = 9  
   x2 – 6x + 9 = 17  
   (x – 3)2 = 17  
   x – 3 =   
   x =   
   (3)
8. The quadratic equation x2 + 2x – 24 = 0 has the same solutions as which of the following equations?  
   x2 + 2x – 24 = 0  
   24 = 24  
   x2 + 2x = 24  
   1 = 1  
   x2 + 2x + 1 = 25  
   (2) (x + 1)2 = 25
9. Find the solutions using completing the square for x2 + 4x – 5 = 0.  
   x2 + 4x – 5 = 0  
   5 = 5  
   x2 + 4x = 5  
   4 = 4  
   x2 + 4x + 4= 9  
   (x + 2)2 = 9  
   x + 2 =   
   x = = 1, -5  
   (1) 1, -5
10. Solve by completing the square for  
    x2 – 12x + 32 = 0.  
      
    x2 – 12x + 32 = 0  
    -32 = -32  
    x2 – 12x = -32  
    36 = 36  
    x2 – 12x + 36 = 4  
    (x – 6)2 = 4  
    x – 6 = = 4, 8  
    (1) 4, 8
11. Which of the following is not a rational number?  
      
    (4) 7 +
12. Show how you arrived at your answers.
13. What are all possible integer solutions for b and p in the equation x2 + bx + 64 = (x + p)2x2 + bx + 64, =   
    b =   
    (x - 8)2 = (x + p)2 => b = -16, p = -8  
    (x + 8)2 = (x + p)2 => b = 16, p = 8
14. Use the completing the square method to find the two solutions to the equation:  
    x2 – 16x – 7 = 0  
    7 = 7  
    x2 – 16x = 7  
    64=64  
    x2 – 16x + 64 = 71  
    (x – 8)2 = 71  
    x – 8 =   
    x =
15. Ramon says that is irrational and that   
    , therefore is irrational. What is wrong with his reasoning process? Explain.  
      
     is an approximation for Therefore:  
    . is a rational number because it involves division involving two rational numbers.  
    Rule: rational rational is always rational.
16. Use the completing the square method to find the two solutions to the equation:  
    x2 – 5x = 14.  
      
    x2 – 5x = 14  
    6.25 = 6.25  
    x2 – 5x + 6.25 = 20.25  
    (x – 2.5)2 = 20.25  
    x – 2.5 =   
    x = =
17. Use the completing the square method to find the two solutions to the equation:  
    x2 + 2ax = b in terms of a and b.  
    x2 + 2ax = b  
    c = c  
    x2 + 2ax + c = b + c  
    c =   
    x2 + 2ax + a2 = b + a2   
    (x + a)2 = x2 + ax + ax + a2 = x2 + 2ax + a2  
    (x + a)2 = b + a2x + a =   
    x =

## 3.4 Solving Quadratic Equations by Factoring

When the quadratic equation contains a trinomial that can be factored, the equation can be solved very quickly. This requires first getting the equation in a form with all the terms on one side of the equation and a zero on the other side of the equation.

### Solving a Quadratic Equation That is Already Factored

(x – 2)(x – 3) = 0  
x – 2 = 0, x = 2  
x – 3 = 0, x = 3

### Solving Equations by First Factoring the Quadratic Trinomial

**Example 6**

Since this polynomial does not have a constant term, it can be factored with the greatest common factor method.

x(x – 7) = 0  
x = 0  
x – 7 = 0, x = 7

### Check Your Understanding of Section 3.4

1. Multiple-Choice

For each equation, find all values of x that satisfy it.

1. (x – 2)(x – 3) = 0  
   **(1) 2, 3**
2. (x – 3(x + 4) = 0  
   **(1) 3, -4**
3. x(x – 5) = 0  
   **(4) 5, 0**
4. x2 + 10x + 24 = 0  
   (x + 6)(x + 4)  
   **(2) -4, -6**
5. x2 + 2x = 15  
   x2 + 2x – 15 = 0  
   (x + 5)(x – 3) = 0  
   **(2) 3, -5**
6. x2 + 6x = 0  
   x(x + 6) = 0  
   **(3) 0, -6**
7. x2 – 6x + 9 = 0  
   (x – 3)2 = 0  
   **(1) 3**
8. 2x2 – 16x + 14 = 0  
   2(x2 – 8x + 7) = 0  
   2(x – 7)(x – 1) = 0  
   **(3) 1, 7**
9. 3x2 + 15x – 108 = 0  
   3(x2 + 5x – 36) = 0  
   Factors: 9, -4  
   3(x + 9)(x – 4) = 0  
   **(2) 4, -9**
10. x2 – 9 = 0  
    (x – 3)(x + 3) = 0  
    **(3) 3, -3**
11. Show how you arrived at your answers.
12. Faith tries to solve the equation x2 – 3x = 0 by first dividing both sides by x to get x – 3 = 0. She concludes that the only solution is x = 3. Is she correct? Explain.  
      
    No. By dividing by x, she is eliminating one solution. By factoring, we get:  
      
    x(x – 3) = 0, x = 0, 3
13. The cubic polynomial x3 – 6x2 + 11x – 6 can be factored into (x – 1)(x – 2)(x – 3). How cn you use this fact to find all solutions to the equation: x3 – 6x2 + 11x – 6?  
      
    The roots of the equation are:   
    x3 – 6x2 + 11x – 6 = 0  
    Therefore: (x – 1)(x – 2)(x – 3) = 0  
    Roots:   
    x – 1 = 0, x = 1  
    x – 2 = 0, x = 2  
    x – 3 = 0, x = 3
14. If ab = 0, and it is known that , what conclusion can you make about b? Explain.  
      
    b must be zero, since the product of ab is zero, and a is not equal to zero. Therefore, it is b that must be zero.
15. The equation x4 – 13x2 + 36 has four solutions. What are they?  
    Factors: -9, -4  
    (x2 – 9)(x2 – 4) = x4 – 4x2 – 9x2 + 36 =   
    Difference of squares formulas  
    x4 – 13x2 + 36  
    (x + 3)(x – 3)(x + 2)(x – 2)  
    x + 3 = 0, x = -3  
    x – 3 = 0, x = 3  
    x + 2 = 0, x = -2  
    x – 2 = 0, x = 2
16. Stephanie tries to solve the equation   
    x2 + 6x = 40 by first factoring the left hand into x(x + 6) = 40, and then concludes that either   
    x = 40, or x + 6 = 40. What is wrong with this reasoning?  
      
    Assuming x = 40, requires assuming x + 6 = 1, which means that x = -5, which contradicts the first assumption.  
      
    Assuming x + 6 = 40, gives x = 34, meaning that x must be 40/34 which contradicts the first assumption.  
      
    Solving the equation means finding the roots once everything to solved for is on the left hand side and the right hand side is zero.  
    x2 + 6x = 40  
    -40 = -40  
    x2 + 6x – 40 = 0  
    (x + 10)(x – 4) = 0  
    x + 10 = 0, x = -10  
    x – 4 = 0, x = 4

## 3.5 The Relationship Between Factors and Roots

### Check Your Understanding of Section 3.5

1. Multiple-Choice
2. What are the roots of the equation   
   (x – 2)(x + 5) = 0?  
   **(2) 2 and -5**
3. What are the roots of the polynomial   
   (x + 4)(x – 7)?  
   **(3) -4, 7**
4. If the roots of an equation are 3 and -6, what could the equation be?  
   **(2) (x – 3)(x + 6) = 0**
5. If the roots of a polynomial are 4 and -2, what could the polynomial be?  
   **(3) (x – 4)(x + 2)**
6. If the factors of a polynomial are (x – 5) and   
   (x – 2), what are the roots of that polynomial?  
   **(1) 5 and 2**
7. If the roots of a polynomial are 1 and -8, what could be the factors?  
   **(1) (x – 1) and (x + 8)**
8. If a polynomial has factors of (x – p) and (x + q), what are the roots of the polynomial?  
   **(4) p and -q**
9. If a cubic polynomial has factors of (x + 2),   
   (x – 3), and (x – 7), what are the roots of the polynomial?  
   **(2) -2, 3, and 7**
10. If a cubic polynomial has roots of 5, 3, and -1, what could the factors of the polynomial be?  
    **(3) (x – 5), (x – 3) and(x + 1)**
11. Which is a root of the equation   
    x3 – 6x2 + 13x – 120 = 0?  
    **(Their solution: (2) 3, does not check).**Plugging 3 into the equation gives a left hand side value of -108.
12. Show how you arrived at your answers.
13. A cubic equation has three roots: 2, -2, and 4. What could the equation be?  
    (x – 2)(x + 2)(x – 4) = 0
14. If an equation has two roots, -4 and -7, what could the equation be?  
    (x + 4)(x + 7) = x2 + 7x + 4x + 28 =  
    (x + 4)(x + 7) = 0 or  
    x2 + 11x + 28 = 0
15. Camilla says the roots of a polynomial are just the factors with the sign changed. Is this accurate? Explain.  
      
    No. Factors still include a variable, but roots are just numbers.
16. If a polynomial has factors (2x + 3) and   
    (2x + 5), what are the two roots?  
    (2x + 3) = 0, x = -1.5  
    (2x + 5) = 0, x = -2.5  
    When there is a coefficient in front of the variable, roots are not found by just changing the sign.
17. The polynomial x2 – 4x – 3 does not seem to factor into (x – p)(x – q) with p and q as integers, but it might factor if p and q don’t have to be integers. By solving the equation   
    x2 – 4x – 3 = 0 by completing the square, it is possible to find the roots. Find the roots and then use them to find the factors.  
    x2 – 4x – 3 = 0  
    3 = 3  
    x2 – 4x = 3  
    4 = 4  
    x2 – 4x + 4 = 7  
    (x – 2)2 = 7  
    x – 2 =   
    x = , x =   
    Factors:  
    (x - 2 - )(x + – 2)

## 3.6 Solving Quadratic Equations with the Quadratic Formula

Many quadratic formulas do not factor. Though completing the square is a technique that will work when factoring is not possible, it is a lengthy process, with many opportunities for careless errors. An efficient way to solve for the roots of a quadratic equation is to use the quadratic formula.

ax2 + bx + c = 0

The above is the same answer you would get if you did completing the square process with the equation:

### Check Your Understanding of Section 3.6

1. Multiple Choice

For each question, find all solutions for the variable using the quadratic formula.

1. x2 – 6x + 8 = 0  
   a = 1, b = -6, c = 8  
   **(1) 2, 4**
2. x2 + 2x – 15 = 0  
   a = 1, b = 2, c = -15
3. x2 + 6x – 16 = 0  
   a = 1, b = 6, c = -16  
   **(2) 2, -8**
4. -x2 - 9x – 20 = 0  
   a = -1, b = -9, c = -20  
      
   **(4) -4, -5**
5. 2x2 + 7x – 4 = 0  
   a = 2, b = 7, c = -4  
   **(1)**
6. x2 + 4x -7 = 0  
   a = 1, b = 4, c = -7  
   **(4)**
7. 6x2 – 13x + 6 = 0  
   a = 6, b = -13, c = 6  
   **(3)**
8. x2 - 2x – 4 = 0  
   a = 1, b = -2, c = -4

**(1)**

1. x2 = x + 1  
   x2 – x – 1 = 0  
   a = 1, b = -1, c = -1  
   **(1)**
2. -16t2 + 64t + 80 = 0  
   -16(t2 - 4t – 5) = 0  
   t2 - 4t – 5 = 0  
   a = 1, b = -4, c = -5  
   **(2) -1, 5**
3. **Show how you arrived at your answers.**
4. Solve for all values of x that satisfy the equation x2 – 10x + 22 = 0. Round Answers to the nearest hundredth.  
   a = 1, b = -10, c = 22  
    **= 3.27, 6.73**
5. If the solution to a quadratic equation   
   ax2 + bx + c = 0, is , what could the values of a, b and c be?  
     
   b = 5  
   2a = 2, a = 1  
   b2 – 4ac = 17  
   52 -4(1)c = 17  
   25 – 4c = 17  
   4c – 17 = 4c – 17  
   25 – 17 = 4c  
   4c = 8, c = 2  
     
   **a = 1, b = 5, c = 2**
6. Use the quadratic formula to solve for both values of x in terms of c for the equation   
   x2 + 4x + c = 0  
   a = 1, b = 4, c = ?
7. Are the solutions to the quadratic formula   
   x2 + 3x – 2 = 0 rational or irrational?  
   a = 1, b = 3, c = -2  
   Irrational. Square root of 17 is an irrational number, so both solutions are irrational.
8. What are the two solutions to the equation   
   x2 + 3 = 4x + 6?  
   -4x – 6 = -4x – 6  
   x2 + 3 – 4x – 6 = 0  
   x2 – 4x – 3 = 0  
   a = 1, b = -4, c = -3

## 3.7 Using the Discriminant to Determine the Number of Unique Roots of a Quadratic Equation

The discriminant of a quadratic equation   
*ax2 + bx + c = 0* is equal to *b2 – 4ac*.

The expression *b2 – 4ac* is called the determinant.

When the discriminant is negative, we say the equation has no *real* solutions, because the square root of a negative is an imaginary number.

If *b2 – 4ac > 0,* there are two unique real solutions.

If *b2 – 4ac = 0,* there is one unique real solution.

If *b2 – 4ac < 0,* there are no unique real solutions.

### Check Your Understanding of Section 3.7

1. Multiple Choice
2. How many unique real solutions does this equation have? x2 + 12x + 10 = 0  
     
   a = 1, b = 12, c = 10  
   *b2 – 4ac =* 122 – 4(1)(10) = 144-40 = 104  
   **(2) 2**
3. How many unique real solutions does this equation have? x2 – 16x + 64 = 0  
     
   a = 1, b = -16, c = 64  
   *b2 – 4ac =* (-16)2 – 4(1)(64) = 256 – 256 = 0  
   **(1) 1**
4. How many unique real solutions does this equation have? x2 + 9x + 13 = 0  
     
   a = 1, b = 9, c = 13  
   *b2 – 4ac =* (9)2 – 4(1)(13) = 81 – 52 = 29  
   **(2) 2**
5. How many unique real solutions does this equation have? 2x2 – 16x + 32  
     
   a = 2, b = -16, c = 32  
   *b2 – 4ac =* (-16)2 – 4(2)(32) = 256 – 256 = 0  
   **(1) 1**
6. How many unique real solutions does this equation have 3x2 + 18x + 30 = 0  
     
   a = 3, b = 18, c = 30  
   *b2 – 4ac =* (18)2 – 4(3)(30) = 324 – 360 = -36  
   **(4) 0**
7. How many unique real solutions does this equation have? 4x2 + 16x – 20 = 0  
     
   a = 4, b = 16, c = -20  
   *b2 – 4ac =* (16)2 – 4(4)(-20) = 256 + 320 = 576  
   **(2) 2**
8. If the equation x2 + 14x + c = 0, has only one unique real solution, what is the value of c?  
     
   a = 1, b = 14, c = c  
   *b2 – 4ac =* (14)2 – 4(1)c = 196 – 4c = 0  
   4c = 196, c = 49  
   **(2) 49**
9. If the equation 3x2 – 30x + c = 0 has only one unique real solution, what is the value of c?  
     
   a = 3, b = -30, c = c  
   *b2 – 4ac =* (-30)2 – 4(3)c = 900 – 12c = 0  
   12c = 900, c = 75  
   **(3) 75**
10. If the equation x2 + bx + 81 has only one unique real solution, what are all possible values of b?  
      
    a = 1, b = b, c = 81  
    *b2 – 4ac =* b2 – 4(1)(81)   
    b2 = 324, b =   
    **(1) [18, -18]**
11. If the equation -2x2 + bx – 50 = 0 has only one unique real solution, what are all possible values of b?  
      
    a = -2, b = b, c = -50  
    *b2 – 4ac =* b2 – 4(-2)(-50) = b2 – 400 = 0  
    b2 = 400, b =   
      
    The answer key is the solution to a different problem, not this one.
12. Show how you arrived at your answers
13. What is the greatest integer value of c that would make the equation x2 -24x + c = 0 have two unique solutions?  
      
    Two real solutions implies discriminant > 0.  
    a = 1, b = -24, c = c  
    *b2 – 4ac =* (-24)2 – 4(1)c = 576 – 4c > 0  
    4c < 576, c < 144, c = 143  
    576 – (4)(143) = 4 > 0
14. What is the least integer value of c that would make the equation x2 + 6x + c = 0 have no real solutions.  
    No real solutions implies discriminant < 0.  
    a = 1, b = 6, c = c  
    *b2 – 4ac =* (-24)2 – 4(1)c = 576 – 4c  
    576 – 4c < 0, 4c > 576, c > 144  
    c = 145  
    576 – 4(1)(145) = -4 < 0
15. In how many points would the line y = 3x – 1 intersect with the parabola y = x2?  
      
    x2 = 3x – 1  
    -3x + 1 = -3x + 1  
    x2 – 3x + 1 = 0  
    a = 1, b = -3, c = 1  
    *b2 – 4ac =* (-3)2 – 4(1)(1) = 9 – 4 = 5  
    The line would intersect the parabola in two points, because the equation has two real solutions.
16. If the line y = 2x + c intersects the parabola   
    y = x2 at only one point, what is the value of c?  
      
    For an intersection at only one point, the discriminant must be zero, for then there would be only one solution.  
    x2 = 2x + c  
    -2x – c = -2x – c  
    x2 - 2x – c = 0  
    a = 1, b = -2, c = -c  
    *b2 – 4ac =* (-2)2 – 4(1)(-c) = 4 + 4c = 0  
    4c = -4, c = -1  
    a = 1, b = -2, c = -1  
    x2 – 2x – (-1) = x2 – 2x + 1 = 0  
    Factoring (x – 1)2 = 0  
    ONLY ONE SOLUTION: x = 1
17. The equations x2 + 6x – 16 = 0 and   
    x2 + 6x – 10 = 0 both have two unique real solutions. Even though the discriminant is positive for both equations, there is something about the value of each discriminant that gives you more information about the solutions. What is it about the value of the discriminant for these equations that explains how the equations differ.

x2 + 6x – 16 = 0  
a = 1, b = 6, c = -16  
*b2 – 4ac =* (6)2 – 4(1)(-16) = 36 + 64 = 100  
  
x2 + 6x – 10 = 0  
a = 1, b = 6, c = -10  
*b2 – 4ac =* (6)2 – 4(1)(-10) = 36 + 40 = 76  
  
Both equations have the same values for a and b, and therefore the only difference is the discriminant portion.  
First equation:

Second equation:

(irrational)

## 3.8 Rationalizing the Denominator

When a fraction has a radical in the denominator, like , the fraction is commonly rewritten as an equivalent fraction that no longer has a radical in the denominator. The process of converting from one form to the other is known as rationalizing the denominator.

Given the choice between an irrational numerator and an irrational denominator, it is preferred to have the denominator be rational.

### Check Your Understanding of Section 3.8

1. Multiple Choice
2. Which expression is equivalent to after the denominator has been rationalized?  
   **(2)**
3. Which expression is equivalent to after the denominator has been rationalized?  
   **(4)**
4. Which expression is equivalent to after the denominator has been rationalized?  
   **(2)**
5. Which expression is equivalent to after the denominator has been rationalized?  
   **(1)**
6. Which expression is equivalent to after the denominator has been rationalized?  
   **(3)**
7. Which expression is equivalent to after the denominator has been rationalized?  
   **(4)**
8. Which expression is equivalent to after the denominator has been rationalized?  
   **(2)**
9. Which expression is equivalent to after the denominator has been rationalized?  
   **(4)**
10. Which expression is equivalent to after the denominator has been rationalized?  
    **(4)**
11. Which expression is equivalent to after the denominator has been rationalized?  
    **(3)**
12. Show how you arrived at your answers.
13. A decimal approximation for is 1.4142. Using this, which would be easier to calculate by hand: or ?  
      
    Generally it is preferable for the numerator to contain the irrational and not the denominator. The second fraction is easier to calculate with long division.
14. Rationalize the denominator of the fraction .  
    (using the product rule for square roots)
15. How can you rationalize the denominator in the expression , which has a fourth root in the denominator?
16. After rationalizing the denominator, a fraction became . What was the original fraction?
17. is the most famous irrational number of all. Do you think it is possible to rationalize the denominator of the fraction ?  
      
    It seems impossible. Rationalizing the denominator is a process used to eliminate radicals (like square roots) from the denominator of a fraction.

## 3.9 Word Problems Involving Quadratic Equations

### Questions Involving the Area of a Rectangle

### Questions Involving the Height of a Projectile

### Questions Involving the Price of an Item

### Check Your Understanding of Section 3.9

1. Multiple Choice
2. A printer wants to put 54 square inches of text into a rectangle on an 8 by 11 sheet of paper. She wants the text to be surrounded by a border of constant width. Which equation could be used to find the width of the border.  
   **(4) (8-2x) (11-2x) = 54**
3. The width of a rectangle is 10 inches longer than its length. If the area of the rectangle is 56 square inches, which equation could be used to determine its length (l)?  
   **(1) l (l + 10) = 56**
4. A 100-foot fence is used to enclose a rectangle land plot. If the area of the land plot is 456 square feet, which equation can be used to determine the length (x) of the land plot.  
   **(3) x (50 – x) = 456**
5. The height of a projectile in feet at time t is determined by the equation   
   h = -16t2 + 128t + 320. At what time will the projectile be 560 feet high?  
   560 = -16t2 + 128t + 320  
   -560 = -560  
   -16t2 + 128t – 240 = 0  
   Divide by -16.  
   t2 - 8t + 15 = 0  
   a = 1, b = -8, c = 15  
   t = 3, 5 seconds  
   **(2) 5 seconds**
6. The height of a projectile in feet at time t is determined by the equation   
   h = -16t2 + 112t + 128. At what time will the projectile be 0 feet high?  
     
   -16t2 + 112t + 128 = 0  
   Divide by -16  
   t2 -7t -8 = 0  
   (t - 8)(t + 1) = 0  
   t = -1, 8 seconds  
   **(4) 8 seconds**
7. The height of a projectile in meters at time t is determined by the equation  
   h = -4.9t2 + 14.7t + 88.2. At what time will the projectile be 68.6 meters?  
     
   68.6 = -4.9t2 + 14.7t + 88.2  
   -68.6 = 68.6  
   -4.9t2 + 14.7t + 19.6 = 0  
   a = -4.9, b = 14.7, c = 19.6  
   ck: -4.9(4)2 + 14.7(4) + 88.2 =   
   -78.4 + 58.8 + 88.2 = 68.6  
   **(2) 4 seconds**
8. When a textbook sells for $20, 70 copies are purchased. Every time the price is increased by $3, the number of people purchasing the book decreases by 8. Which equation can be used to determine the number of $3 increases needed to make the total revenue from the books equal $1,426?  
     
   x = number of $3 increases  
   **(4) (20 + 3x)(70 – 8x) = 1426**
9. When the cost of a movie is $10, 200 people buy tickets. Each time the price of the movie is increased by $3, the number of people buying tickets decreases by 20. Which equation can be used to determine the number of $3 increases needed to make the total revenue from the movie tickets equal to $2,660?  
   x = number of $3 ticket price increases  
   **(3) (10 + 3x) (200 – 20x) = 2660**
10. When a candy bar costs $0.50, 500 kids buy it. Each time the price increases by $0.05, the number of kids buying it decreases by 10. Which equation could be used to determine the number of 0.05 increases needed to make the total revenue from the candy equal to $450?  
      
    x = the number of $0.05 price increases  
    **(1) (0.5 + 0.05x) (500 – 10x) = 450**
11. The surface area of a box with a square base is S =2l2 + 4hl, where l is the length side of the square base, and h is the height of the box. If the height of the box is 5 feet, which equation could be used to find l to make the surface area 238 feet?  
      
    S =2l2 + 4hl = 2l2 + 4(5)l  
    **(4) 2l2 + 20l = 238**
12. Show how you arrived at your answers.
13. The height, in feet, of a projectile after t seconds is determined by the equation  
    h = -16t2 + 128t + 144. At what two times is the projectile exactly 384 feet high?  
      
    -16t2 + 128t + 144 = 384  
    -384 = -384  
    -16t2 + 128t - 240 = 0  
    Divide by -16  
    t2 – 8t + 15 = 0  
    (t – 3)(t – 5) = 0  
    t = 3, 5
14. A 4-inch by 6-inch photo is put into a picture frame with a border of constant width. If the area of the frame, including the picture is 80 square inches, find the equation for determining the width of the border and use that equation to solve for the width.  
      
    (2x + 4) (2x + 6) = 80  
    (2x)(2x) + (2x)(6) + (4)(2x) + (4)(6) =  
    4x2 + 12x + 8x + 24 = 4x2 + 20x + 24  
    4x2 + 20x + 24 = 80  
    -80 = -80  
    4x2 + 20x – 56 = 0  
    Divide by 4  
    x2 + 5x - 14 = 0  
    (x + 7) ( x – 2 ) = 0  
    x = 2 (border must be positive number)  
    **Width = 2x + 4 = 8**  
    Length = 2x + 6 = 10
15. When a song costs $0.99, it gets downloaded 200,000 times. Each time the prices is increased by $0.10, the number of downloads decreases by 10,000. How many $0.10 increases need to happen for the total revenue to be $223.500? Create an equation and solve.  
    x = number of times the price increases by $0.10  
    R = (0.99 + 0.10x)(200000 – 10000x) = 223500  
    198000 – 9900x + 20000x – 1000x2 = 223500  
    -223500 = -223500  
    -1000x2 + 10100x – 25500 = 0  
    Divide by -1000  
    x2 – 10.1 x + 25.5 = 0  
    a = 1, b = -10.1, c = 25.5
16. A man is 35 years older than his son. The product of their ages is 884. Write an equation that can be used to solve for the mans age.  
      
    x = man’s age  
    x (x – 35) = 884  
    x2 – 35x = 884  
    -884 = -884  
    x2 – 35x - 884 = 0  
    a = 1, b = -35, c = -884
17. The perimeter of a rectangle is 44 inches. The area of the rectangle is 120 square inches. If the width of the rectangle is x inches, determine the length of the rectangle using an algebraic solution.  
    l = length of the rectangle

x = width of the rectangle = (44/2 – l) = 22- l  
l (22 – l) = 120  
22l – l2 = 120  
l2 – 22l = l2 – 22l  
l2 – 22l + 120 = 0  
a = 1, b = -22, c = 120