**Barron’s Let’s Review Regents – Algebra I**

# Chapter 3: Quadratic Equations

## 3.1 Solving Quadratic Equations by Taking the Square Root of Both Sides of the Equation

An equation that has a variable raised to the second power is called a quadratic equation. Quadratic equations require more advanced techniques to solve. The simplest quadratic equations to solve are the ones where there is no x-term, like:   
x2 = 25 or x2 + 2 = 38.

Examples:   
x2 = 9  
x2 + 5x = 24  
x2 + 5x + 6 = 0

### One-Step Quadratic Equations

If a quadratic equation is in the form x2 = c, the exponent can be eliminated by taking the square root of both sides. There are two answers, generally, since positive times positive is a positive whereas negative times negative also equals a positive.

**Example 1**

Solve for all values of x that satisfy the equation of   
x2 = 25.

x2 = 25  
 =   
x = , x = 5 or x = -5

**Example 2**

x2 = 26  
 =   
x =

### Two-Step Quadratic Equations with Constants

**Example 3**

x2 + 5 = 54  
-5 = -5  
x2 = 49  
 = =

**Example 4**

Solve for all values of x that satisfy the equation   
x2 + 9 = 36.

x2 + 9 = 36  
-9 = -9  
x2 = 27  
 =   
x = , = +

**Example 5**

Solve for all values of x that satisfy the equation:  
(x + 1)2 = 25  
 =   
x + 1 =   
-1 = -1  
x =   
x = 4, x = -6

**Example 6**

Solve for all values of x that satisfy the equation:  
(x + 3)2 = 17  
 = 17  
x + 3 =   
-3 = -3  
x = -3   
x = -3 + or x = -3 -

**Example 7**

Solve for all values of x that satisfy the equation:  
x2 + 6x + 9 = 17

x2 + 6x + 9 = 17  
Factors: **(3,3)**, (-3,-3), (1,9), (-1, -9)  
(x + 3)2 = 17  
x + 2 =   
-2 = -2  
x = -2 + , or x = -2 -

### Two-Step Quadratic Equations with Coefficients

Solve 2x2 = 50.

**Example 8**

Solve for x in terms of a and b in the equation ax2 = b.

ax2 = b

**Example 9**

The formula for the volume of a cylinder is . Solve this equation for r in terms of V, h, and .

Since r represents the radius of the cylinder, the negative answer can be disregarded.

### Solving Square Root Equations

x = 9

x – 2 = 25  
2 = 2  
x = 27

If the square root sign is not already isolated, first isolate it with algebra.

-3 = -3  
x = 25

### Check Your Understanding of Section 3.1

1. Multiple Choice
2. Find all solutions to x2 = 36.  
   **(4) 6, -6**
3. Find all solutions to x2 = 37.  
   **(1)**
4. Find all solutions to x2 - 9 = 40.  
   **(3) 7, -7**
5. Find all solutions to (x + 2)2 = 64.  
   **(2) 6, -10**
6. Find all solutions to (x – 3)2 = 13.  
   **(3)**
7. Find all solutions to (x + 2)2 = 17.  
   **(4)**
8. Find all solutions to 3x2 = 108.  
   x2 = 36  
   (4) 6, -6
9. Find all solutions to 4x2 + 3 = 103.  
   x2 = 25,   
   **(3) 5, -5**
10. Solve for x in terms of c and d: cx2 = d  
    **(2)**
11. Solve for x in terms of g and h:  
    (x + g)2 = h  
    (1)
12. Show how you arrived at your answers.
13. Find all solutions to the equation x2 = 15, rounded to the nearest hundredth.  
    Use calculator:
14. The left-hand side of the equation   
    x2 + 10x + 25 = 64 can be factored into   
    (x + 5)2. Show how you can find the two solutions to this equation by first factoring the left hand side of the equation.  
    -5 = -5
15. If the area of the largest square in this diagram (composed of the two smaller squares and two rectangles) is 49 square units, what is the length of segment x?  
    -2 = -2  
    x = 5  
    Negative results can be disregarded because the length must be a positive number.
16. Diana solves the equation (x + 4)2 = 97 as follows:  
    (x + 4)2 = 97  
    x2 + 16 = 97  
    Wrong approach, incorrect squaring.
17. Find the solution to the equation (x + 1)3 = 8.  
    (x + 1)3 = 8  
    x + 1 = 2  
    x = 1

## 3.2 Solving Quadratic Equations by Guess and Check

There are several ways to solve a quadratic equation like x2 + 10x = 39. For some examples, when the answer is an integer, it is possible to find the answer through guess and check.

Factors: **(-3,13),** (3, -13)

x2 + 10x – 39 = 0  
(x -3)(x+13) = 0  
x = 3, -13

Guess and check is useful if the quadratic equation is a multiple choice question so there are only four things to check.

**Example 1**

Which of the four choices is a solution to the equation x2 + 5x -3 = 33?  
42 + (5)(4) – 3 = 16 + 20 – 3 = 33. Correct.  
**(3) 4**

**Math Facts**

The solutions to a quadratic equation are sometimes called the *roots* or the *zeros* of the equation.

**Example 2**

Which of the four choices is a root of the equation  
x2 – 6x + 4 = 0?

This requires a calculator.

22.391824 - 28.392 + 4 = - 2  
  
27.41674321 - 31.4166 + 4 = 0.0001

Answer:

### Check Your Understanding of Section 3.2

1. Multiple Choice

For each question, use your calculator to check which answer satisfies the quadratic equation:

1. x2 – 8x + 15 = 0  
   (3)(3) – (8)(3) + 15 = 9 – 24 + 15 = 0  
   **(1) 3**
2. x2 – 2x – 8 = 0  
   (1)(1) – (2)(1) – 8 = 1 – 2 – 8 = -9  
   (2)(2) – (2)(2) – 8 = 4 – 4 – 8 = -8  
   (3)(3) – (2)(3) – 8 = 9 – 6 – 8 = -5  
   (4)(4) – (2)(4) – 8 = 16 – 8 – 8 = 0  
     
   **(4) 4**
3. x2 + 5x – 6 = 0  
   (6)(6) + (5)(6) – 6 = 36 + 30 – 6 = 60  
   (-6)(-6) + (5)(-6) – 6 = 36 – 30 – 6 = 0  
     
   **(2) -6**
4. 2x2 + 5x – 3 = 0  
   (2)(1/4)(1/4) + (5)(1/4) – 3 = (2/16) + (5/4) – 3 = (1/8) + (10/8) – 3 = -1/8  
   (2)(1/3)(1/3) + (5)(1/3) – 3 = (2/9) + (5/3) – 3 = (2/9) + (15/9) – 3 = (17/9) – 3 = -10/9  
   (2)(1/2)(1/2) + (5)(1/2) – 3 = (2/4) + (5/2) – 3 = (1/2) + (5/2) – 3 = (6/2) – 3 = 3 – 3 = 0  
   **(3) (1/2)**
5. x2 + 7x = 30  
   (1)(1) + (7)(1) = 8 30  
   (3)(3) + (7)(3) = 9 + 21 = 30  
     
   **(2) 3**
6. x2 – 2x – 2 = 0  
   6.472 – 2 = 1.9997  
   **(4)**
7. x2 + 5x = 6  
     
   (-6)(-7) + (5)(-7) = 42 – 35 = 7 6  
   (-6)(-6) + (5)(-6) = 36 -30 = 6  
   **(2) -6**
8. x2 + 10x + 13 = 0  
   -5 + = -1.5359  
   -4 + = -0.5359  
   -3 + = 0.4641  
   -2 + = 1.4641  
   **(1) -5 +**
9. x2 + 4x = 25  
   x2 + 4x – 25 = 0  
   **(2) -2 -**
10. x2 – 12x – 14 = 0  
    **(4) 6 +**

## 3.3 Solving Quadratic Equations by Completing the Square

Completing the square is a technique for solving quadratic equations that relies on the concept of a perfect square trinomial. It is not the quickest method to solve certain quadratic equations, but it can be used for all quadratic equations.

A perfect square trinomial, like x2 + 6x + 9, is one where the constant term, the 9, is equal to the square of half the coefficient of the x, in this case, the 6.

If the left-hand side of a quadratic equation is already a perfect square, you can factor it and solve using the technique in Section 3.1, Example 7.

If the left-hand side is not a perfect square trinomial, it is possible to make it into one by adding to both sides of the equation. For the equation  
 , you look at the left-hand side and ask, “What constant would make the left-hand side the equation into a perfect square trinomial?” Since is 25, would be a perfect square trinomial.

Add 25 to both sides and complete the question:

You can check these answers by first substituting 3 for both x’s in the equation and then substituting -13 for both x’s in the equation and making sure that they evaluate to 39.

**Example 1**

What are the roots of the equation x2 + 12x = 28?

Example 2

What are the roots of the equation x2 + 4x - 21

**Rational and Irrational Numbers**

A rational number is one whose decimal expansion either terminates or has a repeating pattern. For example, ½ = 0.5 and 4/7 = 0.571428571428 … are rational numbers. An irrational number does not repeat or have a repeating pattern. For example, .’’

**Combining Rational Numbers with Rational Numbers**

When you add, subtract, multiply, or divide a rational number by a rational number, the result is always another rational number.

**Combining Rational Numbers with Irrational Numbers**

When you combine irrational numbers with non-zero rational numbers, the result is always an irrational number.

**Combining Irrational Numbers with Irrational Numbers**

When you combine an irrational number with another irrational number, the result is usually, but not always, an irrational number.

However, it is possible to combine two irrational numbers and get a rational result.

### Check Your Understanding of Section 3.3

1. Multiple Choice
2. x2 + 12x + c is a perfect square trinomial. What is the value of c?  
   (1) 36
3. x2 + bx + 49 is a perfect square trinomial. What is one possible value of b?  
   (2) 14 (also -14)
4. Use completing the square to find both solutions for x in the equation x2 + 8x + 16 = 9.  
   (x + 4)2 = 9  
   x + 4 =   
   x = = -1, -7  
   (1) -1, -7
5. Use completing the square to find both solutions for x in the equation x2 + 10x = 24.  
   x2 + 10x = 24  
   25 = 25  
   x2 + 10x + 25 = 49  
   (x + 5)2 = 49  
   x + 5 = x = -5 = 2, -12  
   (4) 2, -12
6. Use completing the square to find both solutions for x in the equation x2 – 18x = 40.  
     
   x2 – 18x = 40  
   81=81  
   x2 – 18x +81 = 121  
   (x – 9)2 = 121  
   x – 9 =   
   x = 9 = -2, 20  
   (2) -2, 20
7. What is one solution to the equation   
   x2 – 6x = 8?  
   9 = 9  
   x2 – 6x + 9 = 17  
   (x – 3)2 = 17  
   x – 3 =   
   x =   
   (3)
8. The quadratic equation x2 + 2x – 24 = 0 has the same solutions as which of the following equations?  
   x2 + 2x – 24 = 0  
   24 = 24  
   x2 + 2x = 24  
   1 = 1  
   x2 + 2x + 1 = 25  
   (2) (x + 1)2 = 25
9. Find the solutions using completing the square for x2 + 4x – 5 = 0.  
   x2 + 4x – 5 = 0  
   5 = 5  
   x2 + 4x = 5  
   4 = 4  
   x2 + 4x + 4= 9  
   (x + 2)2 = 9  
   x + 2 =   
   x = = 1, -5  
   (1) 1, -5
10. Solve by completing the square for  
    x2 – 12x + 32 = 0.  
      
    x2 – 12x + 32 = 0  
    -32 = -32  
    x2 – 12x = -32  
    36 = 36  
    x2 – 12x + 36 = 4  
    (x – 6)2 = 4  
    x – 6 = = 4, 8  
    (1) 4, 8
11. Which of the following is not a rational number?  
      
    (4) 7 +
12. Show how you arrived at your answers.
13. What are all possible integer solutions for b and p in the equation x2 + bx + 64 = (x + p)2x2 + bx + 64, =   
    b =   
    (x - 8)2 = (x + p)2 => b = -16, p = -8  
    (x + 8)2 = (x + p)2 => b = 16, p = 8
14. Use the completing the square method to find the two solutions to the equation:  
    x2 – 16x – 7 = 0  
    7 = 7  
    x2 – 16x = 7  
    64=64  
    x2 – 16x + 64 = 71  
    (x – 8)2 = 71  
    x – 8 =   
    x =
15. Ramon says that is irrational and that   
    , therefore is irrational. What is wrong with his reasoning process? Explain.  
      
     is an approximation for Therefore:  
    . is a rational number because it involves division involving two rational numbers.  
    Rule: rational rational is always rational.
16. Use the completing the square method to find the two solutions to the equation:  
    x2 – 5x = 14.  
      
    x2 – 5x = 14  
    6.25 = 6.25  
    x2 – 5x + 6.25 = 20.25  
    (x – 2.5)2 = 20.25  
    x – 2.5 =   
    x = =
17. Use the completing the square method to find the two solutions to the equation:  
    x2 + 2ax = b in terms of a and b.  
    x2 + 2ax = b  
    c = c  
    x2 + 2ax + c = b + c  
    c =   
    x2 + 2ax + a2 = b + a2   
    (x + a)2 = x2 + ax + ax + a2 = x2 + 2ax + a2  
    (x + a)2 = b + a2x + a =   
    x =

## 3.4 Solving Quadratic Equations by Factoring

When the quadratic equation contains a trinomial that can be factored, the equation can be solved very quickly. This requires first getting the equation in a form with all the terms on one side of the equation and a zero on the other side of the equation.

### Solving a Quadratic Equation That is Already Factored

(x – 2)(x – 3) = 0  
x – 2 = 0, x = 2  
x – 3 = 0, x = 3

### Solving Equations by First Factoring the Quadratic Trinomial

**Example 6**

Since this polynomial does not have a constant term, it can be factored with the greatest common factor method.

x(x – 7) = 0  
x = 0  
x – 7 = 0, x = 7

### Check Your Understanding of Section 3.4

1. Multiple-Choice

For each equation, find all values of x that satisfy it.

1. (x – 2)(x – 3) = 0  
   **(1) 2, 3**
2. (x – 3(x + 4) = 0  
   **(1) 3, -4**
3. x(x – 5) = 0  
   **(4) 5, 0**
4. x2 + 10x + 24 = 0  
   (x + 6)(x + 4)  
   **(2) -4, -6**
5. x2 + 2x = 15  
   x2 + 2x – 15 = 0  
   (x + 5)(x – 3) = 0  
   **(2) 3, -5**
6. x2 + 6x = 0  
   x(x + 6) = 0  
   **(3) 0, -6**
7. x2 – 6x + 9 = 0  
   (x – 3)2 = 0  
   **(1) 3**
8. 2x2 – 16x + 14 = 0  
   2(x2 – 8x + 7) = 0  
   2(x – 7)(x – 1) = 0  
   **(3) 1, 7**
9. 3x2 + 15x – 108 = 0  
   3(x2 + 5x – 36) = 0  
   Factors: 9, -4  
   3(x + 9)(x – 4) = 0  
   **(2) 4, -9**
10. x2 – 9 = 0  
    (x – 3)(x + 3) = 0  
    **(3) 3, -3**
11. Show how you arrived at your answers.
12. Faith tries to solve the equation x2 – 3x = 0 by first dividing both sides by x to get x – 3 = 0. She concludes that the only solution is x = 3. Is she correct? Explain.  
      
    No. By dividing by x, she is eliminating one solution. By factoring, we get:  
      
    x(x – 3) = 0, x = 0, 3
13. The cubic polynomial x3 – 6x2 + 11x – 6 can be factored into (x – 1)(x – 2)(x – 3). How cn you use this fact to find all solutions to the equation: x3 – 6x2 + 11x – 6?  
      
    The roots of the equation are:   
    x3 – 6x2 + 11x – 6 = 0  
    Therefore: (x – 1)(x – 2)(x – 3) = 0  
    Roots:   
    x – 1 = 0, x = 1  
    x – 2 = 0, x = 2  
    x – 3 = 0, x = 3
14. If ab = 0, and it is known that , what conclusion can you make about b? Explain.  
      
    b must be zero, since the product of ab is zero, and a is not equal to zero. Therefore, it is b that must be zero.
15. The equation x4 – 13x2 + 36 has four solutions. What are they?  
    Factors: -9, -4  
    (x2 – 9)(x2 – 4) = x4 – 4x2 – 9x2 + 36 =   
    Difference of squares formulas  
    x4 – 13x2 + 36  
    (x + 3)(x – 3)(x + 2)(x – 2)  
    x + 3 = 0, x = -3  
    x – 3 = 0, x = 3  
    x + 2 = 0, x = -2  
    x – 2 = 0, x = 2
16. Stephanie tries to solve the equation   
    x2 + 6x = 40 by first factoring the left hand into x(x + 6) = 40, and then concludes that either   
    x = 40, or x + 6 = 40. What is wrong with this reasoning?  
      
    Assuming x = 40, requires assuming x + 6 = 1, which means that x = -5, which contradicts the first assumption.  
      
    Assuming x + 6 = 40, gives x = 34, meaning that x must be 40/34 which contradicts the first assumption.  
      
    Solving the equation means finding the roots once everything to solved for is on the left hand side and the right hand side is zero.  
    x2 + 6x = 40  
    -40 = -40  
    x2 + 6x – 40 = 0  
    (x + 10)(x – 4) = 0  
    x + 10 = 0, x = -10  
    x – 4 = 0, x = 4

## 3.5 The Relationship Between Factors and Roots

### Check Your Understanding of Section 3.5

1. Multiple-Choice
2. What are the roots of the equation   
   (x – 2)(x + 5) = 0?  
   **(2) 2 and -5**
3. What are the roots of the polynomial   
   (x + 4)(x – 7)?  
   **(3) -4, 7**
4. If the roots of an equation are 3 and -6, what could the equation be?  
   **(2) (x – 3)(x + 6) = 0**
5. If the roots of a polynomial are 4 and -2, what could the polynomial be?  
   **(3) (x – 4)(x + 2)**
6. If the factors of a polynomial are (x – 5) and   
   (x – 2), what are the roots of that polynomial?  
   **(1) 5 and 2**
7. If the roots of a polynomial are 1 and -8, what could be the factors?  
   **(1) (x – 1) and (x + 8)**
8. If a polynomial has factors of (x – p) and (x + q), what are the roots of the polynomial?  
   **(4) p and -q**
9. If a cubic polynomial has factors of (x + 2),   
   (x – 3), and (x – 7), what are the roots of the polynomial?  
   **(2) -2, 3, and 7**
10. If a cubic polynomial has roots of 5, 3, and -1, what could the factors of the polynomial be?  
    **(3) (x – 5), (x – 3) and(x + 1)**
11. Which is a root of the equation   
    x3 – 6x2 + 13x – 120 = 0?  
    **(Their solution: (2) 3, does not check).**Plugging 3 into the equation gives a left hand side value of -108.
12. Show how you arrived at your answers.
13. A cubic equation has three roots: 2, -2, and 4. What could the equation be?  
    (x – 2)(x + 2)(x – 4) = 0
14. If an equation has two roots, -4 and -7, what could the equation be?  
    (x + 4)(x + 7) = x2 + 7x + 4x + 28 =  
    (x + 4)(x + 7) = 0 or  
    x2 + 11x + 28 = 0
15. Camilla says the roots of a polynomial are just the factors with the sign changed. Is this accurate? Explain.  
      
    No. Factors still include a variable, but roots are just numbers.
16. If a polynomial has factors (2x + 3) and   
    (2x + 5), what are the two roots?  
    (2x + 3) = 0, x = -1.5  
    (2x + 5) = 0, x = -2.5  
    When there is a coefficient in front of the variable, roots are not found by just changing the sign.
17. The polynomial x2 – 4x – 3 does not seem to factor into (x – p)(x – q) with p and q as integers, but it might factor if p and q don’t have to be integers. By solving the equation   
    x2 – 4x – 3 = 0 by completing the square, it is possible to find the roots. Find the roots and then use them to find the factors.  
    x2 – 4x – 3 = 0  
    3 = 3  
    x2 – 4x = 3  
    4 = 4  
    x2 – 4x + 4 = 7  
    (x – 2)2 = 7  
    x – 2 =   
    x = , x =   
    Factors:  
    (x - 2 - )(x + – 2)

## 3.6 Solving Quadratic Equations with the Quadratic Formula

Many quadratic formulas do not factor. Though completing the square is a technique that will work when factoring is not possible, it is a lengthy process, with many opportunities for careless errors. An efficient way to solve for the roots of a quadratic equation is to use the quadratic formula.

ax2 + bx + c = 0

The above is the same answer you would get if you did completing the square process with the equation:

### Check Your Understanding of Section 3.6

1. Multiple Choice

For each question, find all solutions for the variable using the quadratic formula.

1. x2 – 6x + 8 = 0  
   a = 1, b = -6, c = 8  
   **(1) 2, 4**
2. x2 + 2x – 15 = 0  
   a = 1, b = 2, c = -15
3. x2 + 6x – 16 = 0  
   a = 1, b = 6, c = -16  
   **(2) 2, -8**
4. -x2 - 9x – 20 = 0  
   a = -1, b = -9, c = -20  
      
   **(4) -4, -5**
5. 2x2 + 7x – 4 = 0  
   a = 2, b = 7, c = -4  
   **(1)**
6. x2 + 4x -7 = 0  
   a = 1, b = 4, c = -7  
   **(4)**
7. 6x2 – 13x + 6 = 0  
   a = 6, b = -13, c = 6  
   **(3)**
8. x2 - 2x – 4 = 0  
   a = 1, b = -2, c = -4

**(1)**

1. x2 = x + 1  
   x2 – x – 1 = 0  
   a = 1, b = -1, c = -1  
   **(1)**
2. -16t2 + 64t + 80 = 0  
   -16(t2 - 4t – 5) = 0  
   t2 - 4t – 5 = 0  
   a = 1, b = -4, c = -5  
   **(2) -1, 5**
3. **Show how you arrived at your answers.**
4. Solve for all values of x that satisfy the equation x2 – 10x + 22 = 0. Round Answers to the nearest hundredth.  
   a = 1, b = -10, c = 22  
    **= 3.27, 6.73**
5. If the solution to a quadratic equation   
   ax2 + bx + c = 0, is , what could the values of a, b and c be?  
     
   b = 5  
   2a = 2, a = 1  
   b2 – 4ac = 17  
   52 -4(1)c = 17  
   25 – 4c = 17  
   4c – 17 = 4c – 17  
   25 – 17 = 4c  
   4c = 8, c = 2  
     
   **a = 1, b = 5, c = 2**
6. Use the quadratic formula to solve for both values of x in terms of c for the equation   
   x2 + 4x + c = 0  
   a = 1, b = 4, c = ?
7. Are the solutions to the quadratic formula   
   x2 + 3x – 2 = 0 rational or irrational?  
   a = 1, b = 3, c = -2
8. What are the two solutions to the equation   
   x2 + 3 = 4x + 6?  
   -4x – 6 = -4x – 6  
   x2 + 3 – 4x – 6 = 0  
   x2 – 4x – 3 = 0  
   a = 1, b = -4, c = -3